

Dynamical Analysis of the 3:1 Resonance in the ν Andromedae System

Carlos E. Chavez, Mauricio Reyes-Ruiz, and Hector Aceves

Instituto de Astronomía, Universidad Nacional Autónoma de México, Apdo. Postal 106, Ensenada, Baja California, 22800 México.

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ABSTRACT

Context. In this paper we study the dynamics of the ν Andromedae planetary system proposed by Curiel et al. (2011). We focus on the study of the 3:1 Mean Motion Resonance (hereafter MMR) between ν Andromedae-d and the recently discovered ν Andromedae-e (hereafter ν And-d and ν And-e).

Aims. Numerical simulations of the dynamics of the four planet system are conducted.

Methods. The previously reported apsidal resonance between ν And-c and ν And-d is confirmed. In addition, we find that ν And-d and ν And-e are also in an apsidal resonance condition.

Results. Our results further indicate that the ν Andromedae planetary system configuration is in the middle of a stability island in the semimajor axis-eccentricity domain. Additionally, we performed numerical integrations of the planetary configuration over 500 Myr and found it to be stable.

Conclusions. We conclude that, within the uncertainties in the value of the orbital parameters, it is likely that ν Andromedae planetary system will remain stable for a long timescale.

Key words. planets and satellites: dynamical evolution and stability -stars: individual: ν And

1. Introduction

About $\sim 10\%$ of the exoplanets known to date are in multiple planet systems (<http://exoplanet.eu>). This is likely to be a lower limit since there is still a strong observational bias towards the detection of planets with short periods. Therefore, some of the extrasolar planets detected so far may have planetary companions not yet detected on distant orbits (e.g. Correia et al. (2009)).

Of the exoplanets discovered in multiple planet systems, several cases have been found to be in mean motion resonances (MMR). This is important because resonances tend to stabilize the orbits of the planets involved, Murray & Dermott (2000) section 8.3. Of the cases discovered to date, the 2:1 mean motion resonance is the most common (e.g. HD 73526, HD 82943, HD 128311, GJ 876, Kepler 9 and HD 37124), but there are other configurations such as the 3:2 MMR in HD 45364, the 3:1 MMR in HD 75732 and the 5:1 in HD 202206. In our Solar System, the 3:2 MMR between Neptune and Pluto (albeit not a *bona-fide* planet) that allows the latter to be in a high eccentricity and high inclination on an orbit that crosses the orbit of Neptune, such that these two objects will never collide (e.g. Varadi (1999)). In the context of the so-called Nice model, it is believed that in the early Solar System, Jupiter and Saturn crossed the 2:1 resonance, leading a significant rearrangement of the general architecture of our planetary system (Tsiganis et al. (2005), Morbidelli et al. (2005) and Gomes et al. (2005)).

It has been pointed out recently by Correia et al. (2009), that the study of the dynamics of mean motion resonances of two or more planets interacting in a system, offers the opportunity to constrain and understand the process of planetary formation

and evolution, since these resonances most probably arise from planetary migration.

The ν Andromedae (hereafter ν And) planetary system was the first multiple, extrasolar planetary system discovered orbiting a solar-type star (Butler et al. 1999). The system is known to harbour three extrasolar planets with masses ranging from 0.69 to 14.57 M_J , with M_J being the mass of Jupiter. Recently, Curiel et al. (2011) have found a fourth planet orbiting ν And named, as is the convention, ν And-e. It is the purpose of the work presented in this paper to analyse the dynamics of this new planet to determine whether it is in MMR with ν And-d and, hence, in a likely stable configuration.

The paper is organized as follows. In Section 2 we review the general characteristics of the central star ν And and the known orbiting planetary system. In Section 3, we perform a global frequency analysis of the two planets possibly involved in the 3:1 resonance, namely, ν And-d and ν And-e. Additionally, we present results on the long term evolution of the system. Section 4 focuses on the apsidal resonance between ν And-c and ν And-d and Section 5 focuses on the apsidal resonance between ν And-d and ν And-e. Finally, we summarize our results and offer our conclusions in section 5.

2. ν Andromedae system characteristics

ν And is a bright F8V star with a mass of 1.3 M_\odot and stellar radius of 1.56 R_\odot (Butler et al. 1999). The distance to the star is estimated to be about 13.47 pc, that is 43.93 lyrs (Perryman et al. 1997). The estimated age of the star is 5 Gyrs (Baliunas et al. 1997), and its rotational period is between 9 and 12 days (Baliunas et al. (1997), Ford et al. (1999)). ν And was the first multiple exoplanetary system detected around a main sequence star. It was reported as being a triple planetary sys-

Send offprint requests to: Carlos E. Chavez
e-mail: carlosepech@astro.unam.mx

Table 1. Properties of the planetary companions in ν And (from Curiel et al. (2011)).

Planet	M (M_J)	a (AU)	e
b	0.6876(44)	0.05922166(20)	0.02150(70)
c	1.981(19)	0.827774(15)	0.2596(79)
d	4.132(29)	2.51329(75)	0.2987(72)
e	1.059(28)	5.24558(67)	0.00536(440)

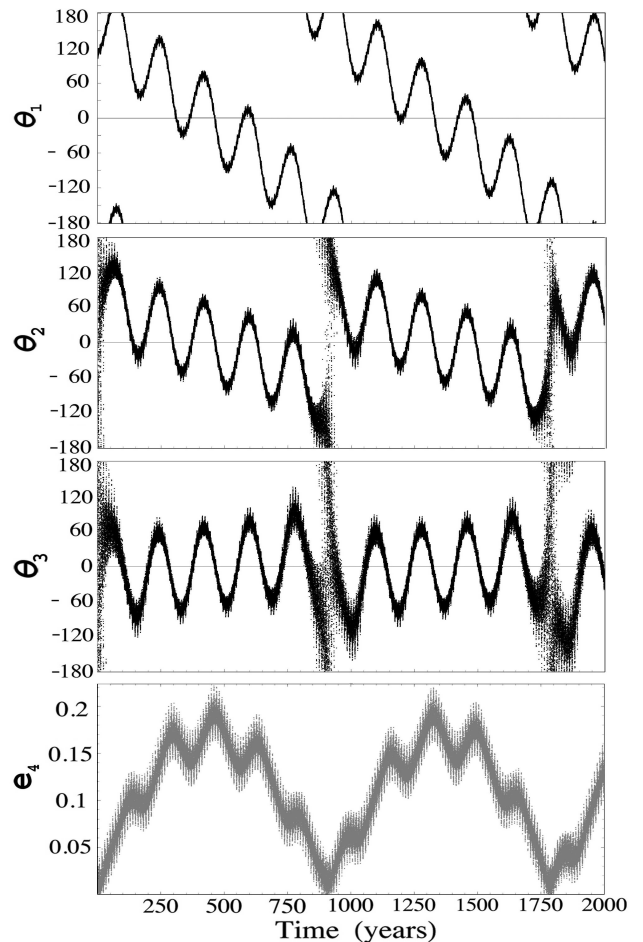
Table 2. Fundamental frequencies calculated from the nominal solution

	Frequency ($^\circ$ /yr)	Period (yr)
n_d	0.285194	1262.3
n_e	0.0947104	3801.06
g_d	0.0000257259	38871.3
g_e	0.00109351	914.484
l_{θ_3}	1.52241	236.467
l_{e_4}	0.415535	866.353

tem by Butler et al. (1999). The estimated masses and orbital parameters for each of the planets around ν And are listed in Table 1. Several studies have been made about the origin and dynamical stability of the triple planetary system (Ford et al. (2005), Chiang et al. (2001), Rivera & Haghighipour (2007) and references therein). The main conclusion of these studies, considering only the first three planets discovered are: Ford et al. (2005) found out that planet–planet scattering with a fourth lost planet could explain the high eccentricities of ν And–c and ν And–d. Chiang et al. (2001) found out that the apsidal resonance between ν And–c and ν And–d is observed for mutual inclinations, between these two planets, smaller than 20° . Rivera & Haghighipour (2007) concluded that there are stability regions for test particles around the 3:1 and 5:1 MMR, and that for $a > 7.5$ AU all test particles were stable for at least 10^7 yrs. They reported additionally that test particles just outside the 1:3 MMR with ν And–d experience large oscillations reaching eccentricities up to the range of 0.2 to 0.3; but these particles are protected from close approaches with ν And–d by the e – ω mechanism. Recently, Curiel et al. (2011) have discovered a fourth planet orbiting the system on the basis of a refined fit for the radial velocity data. Its properties are also described in Table 1. Curiel et al. (2011) propose that the system is close to a 3:1 MMR resonance. The initial estimate for the period of the fourth planet (ν And–e) is 3848.86 days, and the period of the third planet (ν And–d) is 1276.46. The ratio between the two periods is 3.02, very close to being in exact resonance. In a previous study, Rivera & Haghighipour (2007) find that there is an island of stability in the semimajor axis–eccentricity parameter domain, just outside the external values corresponding to the 3:1 MMR. Rivera & Haghighipour (2007) use a large collection of test particles to sample the possible location of new planets.

3. Orbital stability

In this section we analyse the dynamical stability of a planetary system characterized by the orbital parameters reported in Table 1. We refer to this set of parameters for the ν And planetary system as the nominal solution. There are different procedures to check the stability of a system using the frequency analysis developed by Laskar (1993) (e.g. Marzari et al. (2005)). We follow the approach taken by Correia et al. (2005), Correia et al. (2009) and Couetdic et al. (2010) to analyse the stability of the planetary configuration.

**Fig. 1.** Time evolution of the three resonant angles for the 3:1 resonance between ν And–d and ν And–e. The angles θ_1 and θ_2 circulate, while θ_3 librates. The bottom panel shows the oscillation in the time evolution of the eccentricity of ν And–e (called here e_4)

3.1. The 3:1 resonance

To test whether the ν And system is trapped or not in the 3:1 MMR resonance, we perform a frequency analysis of the nominal solution computed over 10 Kyr. Since the orbit of the inner planets of the system is well constrained, we fix the orbital parameters of ν And–b, ν And–c and ν And–d according to Table 1. We also assume the orbits of all planets are coplanar. A series of initial conditions for the eccentricity (e) and semi-major (a) axis of planet e are constructed to test the stability of the region around the nominal solution.

For each initial condition, the orbits of the planets are integrated over 10 kyr using the hybrid integrator included in the *Mercury 6* code (Chambers 1999). For most of the integration, *Mercury* uses a mixed-variable symplectic integrator (Wisdom & Holman 1991) with a time step approximately equal to a fiftieth ($\approx 1/50$) of the Keplerian orbital period of the closest planet (in this case ν And–b). During close encounters, *Mercury* uses a Bulirsch-Stoer integrator with an accuracy parameter of 10^{-12} . The stability of the orbit corresponding to each set of (a, e) values is measured using the frequency analysis introduced by Laskar (1990) and Laskar (1993). According to Laskar (1993) the difference (D) in the value of the fundamental frequency of the motion of the planet under consideration, obtained over two

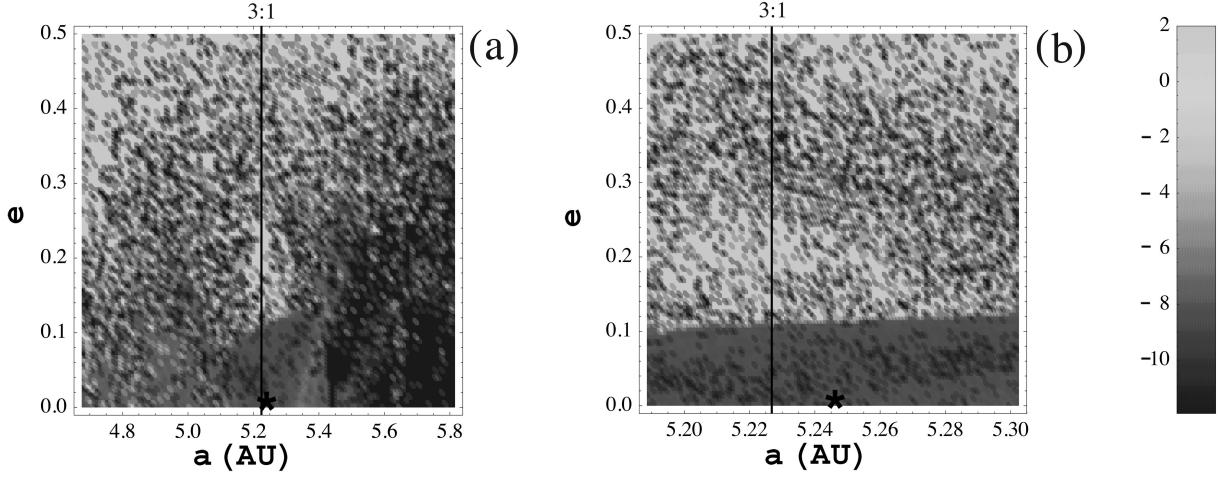


Fig. 2. Stability analysis of the ν And system as a function of semi-major axis, a , and eccentricity for planet-e, e . The logarithm of the stability parameter D is shown on a gray scale map. The lighter colors correspond to the more unstable systems. The log-value +2 for the parameter D corresponds to the ejection of planet-e. The \star symbol corresponds to the nominal solution reported by Curiel et al. (2011). Panel b) is a close-up, in a , of the wider domain shown in panel a) of the parameter space around the nominal solution.

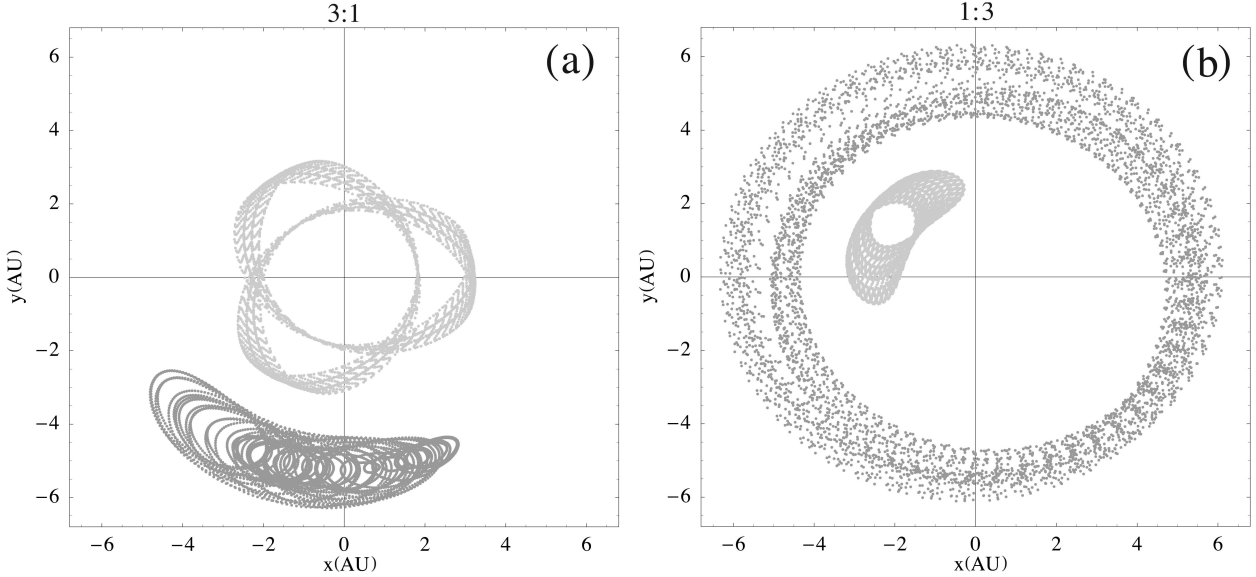


Fig. 3. Orbits in rotating reference frames for 500 yrs for the ν And system. Panel (a) shows the reference frame with the angular velocity M_c in which the 3:1 MMR resonance is observed. Panel (b) shows the same points but in a reference frame that is rotating with M_d , where we observe the 1:3 MMR resonance

consecutive time intervals is a measure of the secular stability of the trajectory.

In addition, we also identify strongly unstable systems as those in which: 1) two planets collide, 2) a planet hits the star (if its astrocentric distance is > 0.005 AU), or 3) a planet is ejected from the system (assumed to occur if the planet travels beyond an astrocentric distance of 100 AU).

The three possible resonant arguments of the 3:1 resonance according to Murray & Dermott (2000) p.491, with $\Omega = 0.0$ because we consider coplanar orbits, are the following:

$$\theta_1 = 3\lambda_e - \lambda_d - 2\omega_d, \quad (1)$$

$$\theta_2 = 3\lambda_e - \lambda_d - \omega_e - \omega_d \quad (2)$$

and

$$\theta_3 = 3\lambda_e - \lambda_d - 2\omega_e, \quad (3)$$

where λ_d , ω_d and λ_e , ω_e are the mean longitude and argument of the pericenter of planet e and d, respectively. Fig. 1 shows the time evolution of the three possible resonant angles and the time evolution of the eccentricity of planet-e, in order to make a direct comparison of its periodicity with the resonant angles. We find that θ_1 and θ_2 are circulating while θ_3 is librating around zero. This behaviour means that the two planets are closer to the resonance defined by θ_3 .

It is clear from Figure 1 that θ_2 and θ_3 have periodic and sudden changes with periods of 866.353 yrs. (we found this period using a Fourier analysis), this period coincides with the period of approximately 866.353 years that can be seen in the oscillation

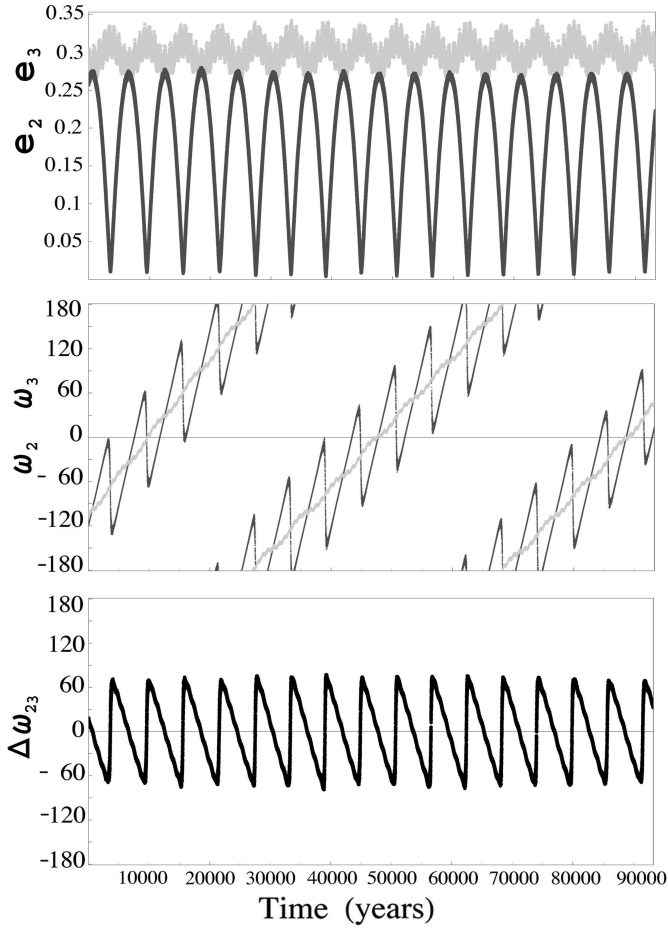


Fig. 4. Apsidal resonance between planet-c and planet-d. The two eccentricities are anti-correlated as shown in the top panel. The eccentricity of planet c (here e_3) gets close to zero periodically. We show in the middle panel the argument of the pericentre of the two planets, it can be noticed that they tend to follow each other. Finally, in the bottom panel we show the difference between the arguments of pericentre ($\Delta\omega_{23}$) of planets c and d. As we can notice this angle is librating around zero.

of the eccentricity of planet-e about zero. This behaviour can be explained by the fact that ω_e is not well defined for circular orbits. Therefore θ_3 is not circulating every 866.353 yrs., but rather librating and every period that planet e comes back to zero this resonant angle suffers the sudden changes observed in Fig. 1.

The fundamental frequencies of the system are the two mean motions (known also as mean angular velocities) $n_d = \frac{d\lambda_d}{dt}$ and $n_e = \frac{d\lambda_e}{dt}$, the two secular frequencies of the pericenter $g_d = \frac{d\omega_d}{dt}$ and $g_e = \frac{d\omega_e}{dt}$, and the libration frequency of the resonant argument $l_\theta = \frac{d\theta_3}{dt}$. The values of each one of these are shown in Table 2.

3.2. Stability analysis

To analyse the stability of our nominal solution, we perform a global frequency analysis (Laskar 1993) in its vicinity (Fig. 2) (a) and (b), similarly as it has been done by Correia et al. (2005), Correia et al. (2009) and Couetdic et al. (2010). For the two planets that are possibly involved in the the resonance (d & e), the system is integrated using a two-dimensional mesh of

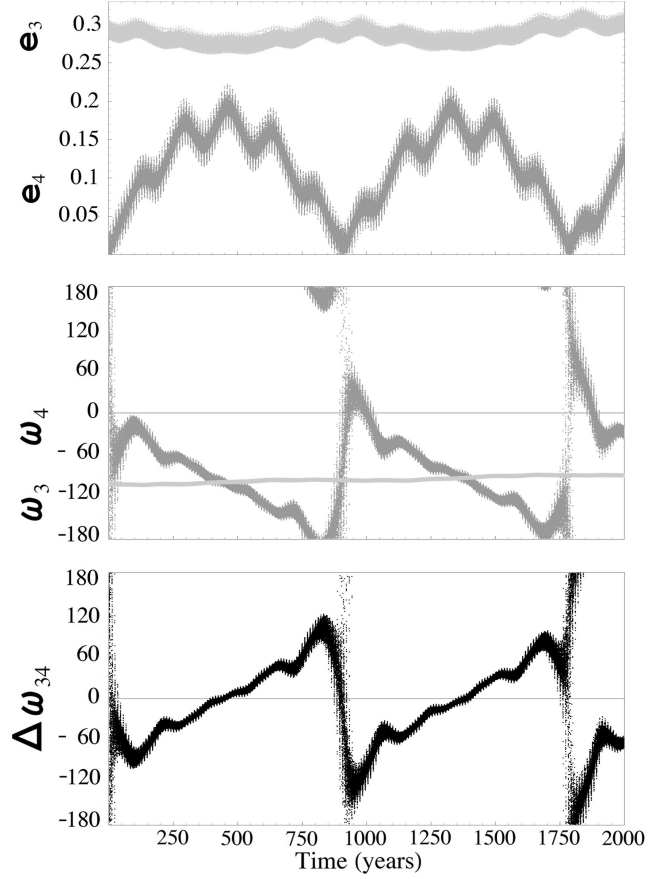


Fig. 5. Apsidal resonance between planet-d and planet-e. The two eccentricities of planets d and e (shown here in light grey and dark grey, respectively) are anti-correlated. The middle panel shows the arguments of pericentre of these two planets, it is possible to notice that they follow each other. Finally, in the bottom panel is shown the difference between these two arguments of pericentre ($\Delta\omega_{34}$), it is possible to notice that this angle is librating around zero.

11,165 initial conditions. We vary the semi-major axis and eccentricity of planet e, and keep the orbital parameters of the other planets at their nominal values.

We integrate each case numerically over 10 kyrs, and then we compute the stability indicator D . This stability indicator is computed by comparing the variation in the measured mean motion over two consecutive $T = 5$ kyr intervals. The parameter D is defined as:

$$D = \frac{|n_e - n'_e|}{T}, \quad (4)$$

where n_e and n'_e are the mean motion of planet e in the first 5 kyr and last 5 kyr of the integration, respectively; the units of D are deg/yr^2 . This parameter D is a measurement of the chaotic diffusion of the trajectory. It should be close to zero for a regular solution and it has a high value for strong chaotic motion (Laskar 1993). We also performed 10 integrations of 1 Myrs and found that in the ν And system, a value of $D < 10^{-7}$ is required for regular motion (comparing the D of stable and unstable integrations).

The left panel of Figure 2 (a) shows the results of our mesh of 11,165 numerical simulations in the domain of semimajor axis, $a \in [4.7, 5.8]$ AU, and eccentricity $e \in [0, 0.5]$. From Fig. 2 (a) we see that there is an island of stability around the nominal solution, shown here with the symbol \star (located at $a = 5.24558$ AU). The region does not seem to contain ejected particles (designated with $D = +2$ in this plot). We show also the location of the 3:1 MMR as a vertical line at $a = 5.22672$ AU. At this scale the location of the nominal solution and the location of the 3:1 MMR are very close to each other. Therefore we decided to zoom-in around the region of the nominal solution, and explore it numerically.

We performed another set of 11,615 integrations around the stability island, focusing on the parameter domain $a \in [5.18, 5.30]$ AU and $e \in [0, 0.5]$. Our results are shown in the right panel of Fig. 2 (b). The shape of the stability island around the 3:1 resonance is easily appreciated on this scale. We notice that most of the systems explored in the a - e mesh are unstable above $e = 0.1$. The nominal solution reported in Curiel et al. (2011) has $a = 5.24558$ AU and $e = 0.00536$ (shown again with \star). That puts the system right in the middle of the stability island. This result support the idea that the solution found by Curiel et al. (2011) is stable and robust, as suggested by these authors. Also, it appears that the only stable zone that exists in the vicinity of the nominal solution is the zone protected by 3:1 resonance.

In Fig. 3 (a and b) we show the evolution of ν And-d and ν And-e over 500 yr in the rotating reference frame of the inner and outer planet, respectively. Since the system is so close to the 3:1 resonance the relative position of the two planets is repeated, and their minimum distance in our simulation was 2.47 AU. The paths of the two planets in this rotating reference frame shows the relationship between the resonance and the frequency of conjunctions (see Murray & Dermott (2000), p. 325). In this particular reference frame, every three orbits of ν And-d corresponds to one orbit of ν And e. Figure 3 also shows the libration of each planet around its equilibrium position. Since there are two other planets on the system ν And, the equilibrium position precesses and does not maintain its path, in the rotating reference frame, for a much longer time.

3.3. Orbital evolution

The previous stability analysis supports the idea that the ν And system is trapped in the island-like stability zone associated with the 3:1 resonance, and that it is very likely to be stable on long term evolution. To test this suggestion we carried out a long term numerical integration of the nominal solution. Using the *Mercury 6* code with the same parameters as described above, we integrated the orbits of the four planets for 500 Myrs. We found that, as suggested by the study of Curiel et al. (2011), these orbits are stable in the long term.

Due to the intense resonant gravitational interaction the three planets involved (c, d & e) exhibit significant changes in their eccentricities, as can be seen in Figs. 4 and 5.

4. Apsidal resonance between ν And-c and ν And-d

In this section we report on the apsidal resonance that was observed in previous research on ν And (e.g. Curiel et al. (2011), Chiang et al. (2001), Rivera & Lissauer (2000), Lissauer & Rivera (2001), Chiang & Murray (2002), Barnes & Quinn (2004), Michtchenko & Malhotra (2004), Nagasawa & Lin (2005), etc).

The secular apsidal resonance is shown in Fig. 4. We find that the eccentricities e_d and e_c are anti-correlated as well, that is, when e_d is maximum e_c is minimum and vice versa. We show ω_c (dark gray) and ω_d (light gray) in the middle panel of Fig. 4. The difference between these two angles is shown in the bottom panel of Fig. 4, it shows that $\Delta\omega_{cd}$ oscillates around zero with an initial amplitude of $\sim 77^\circ$ and a short period of 6×10^3 yrs. Therefore showing that planets c and d are indeed in apsidal resonance.

The physics of the apsidal resonance is the following. The eccentricity of ν And-c and ν And-d are quite large ($e_c = 0.2596$ and $e_d = 0.2987$, Table 1 of (Curiel et al. 2011)) and their semi-major axis are not too different from each other ($a_c = 0.827774$ AU and $a_d = 2.51329$ AU). In principle, they could be as close to each other as $D_{\min} = a_d(1 - e_d) - a_c(1 + e_c) = 0.719906$ AU, but this never happens. As it is possible to notice in Fig. 4, when the two orbits are aligned (that is $\Delta\omega_{cd} = 0.0$) the eccentricity of planet-d is maximum, but at the same time the eccentricity of planet-c is reduced to values $e_c \sim 0$. This helps to avoid close encounters between the two planets, hence helping to stabilize the system, then the two planets never get closer than 0.9 AU.

5. Apsidal resonance between ν And-d and ν And-e

In addition to planet-c and planet-d being in apsidal resonance, we find that planet-d and planet-e are in apsidal resonance as well. This was suggested by Curiel et al. (2011), and here we give the details about this resonance for the first time. Figure 5 shows this apsidal resonance in action. We find that e_d and e_e are anti-correlated. We also find that the maximum eccentricity that planet-e can achieve is quite big ($e_{\max} = 0.233105$). Therefore, in principle, it is possible that planet-e and planet-d can get close to each other $D_{\min} = a_d(1 - e_d) - a_e(1 + e_e) = 2.051726$ AU, but owing to the apsidal resonance, they never get closer than $D_{\min} = 2.47421$ AU.

6. Summary and conclusions

We have carried out a stability analysis for the 4-planet system around ν And by means of a frequency analysis Laskar (1993) and long term numerical simulations. On the basis of our results, we find evidence that ν And-d and ν And-e are in a 3:1 mean motion resonance, as suggested by previous studies. The nominal solution found by Curiel et al. (2011) is located in the middle of a island of stability in the a - e parameter space.

The ν And system is found to be a rich dynamical system, with three of the planets discovered to date interacting strongly via resonances. As described in the paper, ν And e interacts via the 3:1 MMR with ν And d, and additionally they are in apsidal resonance. This prevents them from having close encounters. On another hand, the apsidal resonance reported by previous authors between ν And-c and ν And-d is still present when ν And-e is considered. This means that the three planets are interacting via apsidal and MMR resonances.

The nominal solution of Curiel et al. (2011) is here proved to be stable for 500 Myrs, this along with our results of the stability analysis done via the global frequency analysis, allows us to conclude that the nominal solution is robust and stable.

Finally the reported eccentricity of ν And-e is at the moment very close to zero ($e = 0.00536$). However, according to our results, it should increase to $e = 0.016$ in approximately 10 years. If it is possible to measure this variation after such period, it will provide an important confirmation of the dynamical properties

described in this work. Our results further predict that the eccentricity of ν And-e will reach its maximum value of 0.2 in around 450 years.

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